MA22S3 Tutorial Sheet 6.[1,2]

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Useful facts:

- To solve the equation $\dot{y} + py = f$ multiply across by an integrating factor

$$\lambda(t) = e^{I(t)} = \exp\left(\int_a^t p(\tau)d\tau\right) \tag{1}$$

  and express the right hand side as the derivative of a product.

- This gives solution

$$y(t) = y(a)e^{-I(t)} + e^{-I(t)}\int_a^t f(\tau)e^{I(\tau)}d\tau \tag{2}$$

- To solve the equation $a\ddot{y} + b\dot{y} + cy = 0$, with a, b and c constants, use an exponential substitution $y = \exp(\lambda t)$ and solve for λ. Usually this will give two solutions λ_1 and λ_2 so

$$y = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \tag{3}$$

Questions

1. (2) Obtain the solution to $\dot{y} - 3y = e^{-t}$ with $y(0) = 1$; since an initial condition is chosen at $t = 0$, choose $a = 0$.
2. (2) Obtain a general solution to $(t+1)\dot{y} + y = (t+1)^2$
3. (2) Obtain the general solution to

$$\ddot{y} + \dot{y} - 2y = 0 \tag{4}$$

4. (2) Obtain the general solution to

$$\ddot{y} + 6\dot{y} + 8y = 0 \tag{5}$$

[1]Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/MA22S3

[2]Including material from Chris Ford, to whom many thanks.