

MA22S3 Tutorial Sheet 4.¹²

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Useful facts:

- The **Fourier integral** or Fourier transform:

$$\begin{aligned}f(t) &= \int_{-\infty}^{\infty} dk \widetilde{f(k)} e^{ikt} \\ \widetilde{f(k)} &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dt f(t) e^{-ikt}\end{aligned}$$

- The Dirac delta function:

$$\int_{-\infty}^{\infty} f(t) \delta(t) dt = f(0)$$

and so

$$\int_{-\infty}^{\infty} f(t) \delta(t - a) dt = f(a)$$

Questions

1. (3) Compute the Fourier transform of $f(t) = te^{-t}$ for $t > 0$ and zero otherwise.
2. (2) Using integration by parts or otherwise, write the Fourier transform of df/dt in terms of the Fourier transform of f .
3. (3) Do the following integrals

(a)

$$\int_{-\infty}^{\infty} \delta(t)(t^2 + 3t + 5) dt \tag{1}$$

(b)

$$\int_{-\infty}^{\infty} \delta(t - \pi/4) \sin t dt \tag{2}$$

(c)

$$\int_{-\infty}^{\infty} \delta(t - 1) \ln t dt \tag{3}$$

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²Including material from Chris Ford, to whom many thanks.