MA22S3 Tutorial Sheet 3.[1][2]

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Useful facts:

- **Exponential with imaginary argument**:

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{1}$$

- **Cosine and sine in terms of exponentials**:

$$\begin{aligned} \cos\theta &= \frac{e^{i\theta} + e^{-i\theta}}{2} \\ \sin\theta &= \frac{e^{i\theta} - e^{-i\theta}}{2i} \end{aligned} \tag{2}$$

- The **Fourier integral** or Fourier transform:

$$\begin{aligned} f(t) &= \int_{-\infty}^{\infty} dk\, \widetilde{f(k)} e^{ikt} \\ \widetilde{f(k)} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, f(t) e^{-ikt} \end{aligned}$$

Questions

1. (4) Compute the Fourier transform of $f(t) = e^{-a|t|}$ where a is a positive constant.
2. (4) Express the following function as a Fourier integral:

$$f(x) = \begin{cases} \cos t & |t| < \frac{\pi}{2} \\ 0 & |t| > \frac{\pi}{2} \end{cases}$$

One way to do the required integral is to split the cosine into exponentials.

[1] Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/MA22S3`

[2] Including material from Chris Ford, to whom many thanks.