

## MA22S3 Tutorial Sheet 2.<sup>12</sup>

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Useful facts:

- **Trigonometric identities:** products

$$\begin{aligned}\cos A \cos B &= \frac{1}{2}[\cos(A - B) + \cos(A + B)] \\ \sin A \sin B &= \frac{1}{2}[\cos(A - B) - \cos(A + B)] \\ \sin A \cos B &= \frac{1}{2}[\sin(A + B) + \sin(A - B)].\end{aligned}\tag{1}$$

- **Trigonometric identities:** double angles

$$\begin{aligned}\sin^2 A &= \frac{1}{2}(1 - \cos 2A) \\ \cos^2 A &= \frac{1}{2}(1 + \cos 2A)\end{aligned}\tag{2}$$

- A function  $f(t)$  has period  $L$  if  $f(t + L) = f(t)$ , it is odd if  $f(-t) = -f(t)$  and even if  $f(-t) = f(t)$ .

- **Exponential with imaginary argument:**

$$e^{i\theta} = \cos \theta + i \sin \theta\tag{3}$$

- A function with period  $2\pi$  has the **Fourier series expansion**

$$f(t) = \sum_{n=-\infty}^{\infty} c_n \exp(int).$$

where

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) \exp(-int) dx$$

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<sup>2</sup>Including material from Chris Ford, to whom many thanks.

## Questions

1. (2) What is the period of  $\sin t \cos t$  and  $\sin^3 t$ .
2. (2) Find the Fourier series for  $\sin^3 t$ ; a quick way to do this is to regard it as a trigonometry problem, rather than a Fourier series problem, that is use the trigonometric identities to express it in terms of sines and cosines, rather than doing all the integrals: so start by writing  $\sin^3 t = \sin^2 t \sin t$  and then write  $\sin^2 t$  in terms of  $\cos 2t$ .
3. (4) Express the following periodic function ( $L = 2\pi$ ) as complex Fourier series

$$f(t) = \begin{cases} 0 & -\pi < t < 0 \\ 1 & 0 < t < a \\ 0 & a < t < \pi \end{cases} \quad (4)$$

where  $a \in (0, \pi)$  is a constant.