MA22S3 Tutorial Sheet 2.[1,2]

20 October 2010

Useful facts:

- **Trignometric identities**: products

$$\begin{aligned} \cos A\cos B &= \frac{1}{2}[\cos(A-B)+\cos(A+B)]\\ \sin A\sin B &= \frac{1}{2}[\cos(A-B)-\cos(A+B)]\\ \sin A\cos B &= \frac{1}{2}[\sin(A+B)+\sin(A-B)]. \end{aligned} \quad (1)$$

- **Trignometric identities**: double angles

$$\begin{aligned} \sin^2 A &= \frac{1}{2}(1-\cos 2A)\\ \cos^2 A &= \frac{1}{2}(1+\cos 2A) \end{aligned} \quad (2)$$

- A function $f(t)$ has period L if $f(t+L)=f(t)$, it is odd if $f(-t)=-f(t)$ and even if $f(-t)=f(t)$.

- **Exponential with imaginary argument**:

$$e^{i\theta}=\cos\theta+i\sin\theta \quad (3)$$

- A function with period 2π has the **Fourier series expansion**

$$f(t)=\sum_{n=-\infty}^{\infty} c_n \exp(int).$$

  where

$$c_n=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\exp(-int)dx$$

[1]Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/MA22S3`

[2]Including material from Chris Ford, to whom many thanks.

Questions

1. (2) What is the period of $\sin t\cos t$ and $\sin^3 t$.

2. (2) Find the Fourier series for $\sin^3 t$; a quick way to do this is to regard it as a trigonometry problem, rather than a Fourier series problem, that is use the trigonometric identities to express it in terms of sines and cosines, rather than doing all the integrals: so start by writing $\sin^3 t=\sin^2 t\sin t$ and then write $\sin^2 t$ in terms of $\cos 2t$.

3. (4) Express the following periodic function ($L=2\pi$) as complex Fourier series

$$f(t)=\begin{cases} 0 & -\pi<t<0\\ 1 & 0<t<a\\ 0 & a<t<\pi \end{cases} \quad (4)$$

   where $a\in(0,\pi)$ is a constant.