MA22S3 Tutorial Sheet 1.[1]

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Useful facts:

- **Trignometric identities**: adding angles

$$\begin{aligned} \sin A \pm B &= \sin A \cos B \pm \cos A \sin B \\ \cos A \pm B &= \cos A \cos B \mp \sin A \sin B. \end{aligned} \tag{1}$$

- **Trignometric identities**: products

$$\begin{aligned} \cos A \cos B &= \frac{1}{2}[\cos(A-B) + \cos(A+B)] \\ \sin A \sin B &= \frac{1}{2}[\cos(A-B) - \cos(A+B)] \\ \sin A \cos B &= \frac{1}{2}[\sin(A+B) + \sin(A-B)]. \end{aligned} \tag{2}$$

- **Trignometric identities**: double angles

$$\begin{aligned} \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A. \end{aligned} \tag{3}$$

- **Periodic**: A function is periodic if for some constant L, $f(t+L) = f(t)$. The smallest such L is called the *period*.

- **Integration by parts**:

$$\int_a^b u dv = uv]_a^b - \int_a^b v du \tag{4}$$

- **Periodic, even and odd** A function $f(t)$ has period L if $f(t+L) = f(t)$, it is odd if $f(-t) = -f(t)$ and even if $f(-t) = f(t)$.

- A function with period L has the **Fourier series expansion**

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n t}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n t}{L}\right).$$

where

$$a_0 = \frac{2}{L}\int_{-L/2}^{L/2} f(t) dt$$

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$$\begin{aligned} a_n &= \frac{2}{L}\int_{-L/2}^{L/2} f(t)\cos\left(\frac{2\pi n t}{L}\right) dt \\ b_n &= \frac{2}{L}\int_{-L/2}^{L/2} f(t)\sin\left(\frac{2\pi n t}{L}\right) dt \end{aligned}$$

Questions

1. (2) Establish that
$$\int_{-\pi}^{\pi} dt\ \sin mt\ \cos nt = 0 \tag{5}$$
for integers n and m.

2. (2) Show by checking whether $f(t) = -f(-t)$ for odd, $f(t) = f(-t)$ for even and neither for neither which of the following are odd, even or neither: $\sin t$, t^3+t, t^3+2t^2 and $|t|$.

3. (4) Find the Fourier series representation of the sawtooth function f defined by $f(t) = t$ for $-\pi < t \leq \pi$ and $f(t+2\pi) = f(t)$.