

MA22S3 Tutorial Sheet 1.¹

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Useful facts:

- **Trigonometric identities:** adding angles

$$\begin{aligned}\sin A \pm B &= \sin A \cos B \pm \cos A \sin B \\ \cos A \pm B &= \cos A \cos B \mp \sin A \sin B.\end{aligned}\tag{1}$$

- **Trigonometric identities:** products

$$\begin{aligned}\cos A \cos B &= \frac{1}{2}[\cos(A - B) + \cos(A + B)] \\ \sin A \sin B &= \frac{1}{2}[\cos(A - B) - \cos(A + B)] \\ \sin A \cos B &= \frac{1}{2}[\sin(A + B) + \sin(A - B)].\end{aligned}\tag{2}$$

- **Trigonometric identities:** double angles

$$\begin{aligned}\sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A.\end{aligned}\tag{3}$$

- **Periodic:** A function is periodic if for some constant L , $f(t+L) = f(t)$. The smallest such L is called the *period*.

- **Integration by parts:**

$$\int_a^b u dv = uv \Big|_a^b - \int_a^b v du\tag{4}$$

- **Periodic, even and odd** A function $f(t)$ has period L if $f(t+L) = f(t)$, it is odd if $f(-t) = -f(t)$ and even if $f(-t) = f(t)$.

- A function with period L has the **Fourier series expansion**

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nt}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{L}\right).$$

where

$$a_0 = \frac{2}{L} \int_{-L/2}^{L/2} f(t) dt$$

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$$\begin{aligned}
a_n &= \frac{2}{L} \int_{-L/2}^{L/2} f(t) \cos\left(\frac{2\pi nt}{L}\right) dt \\
b_n &= \frac{2}{L} \int_{-L/2}^{L/2} f(t) \sin\left(\frac{2\pi nt}{L}\right) dt
\end{aligned}$$

Questions

1. (2) Establish that

$$\int_{-\pi}^{\pi} dt \sin mt \cos nt = 0 \quad (5)$$

for integers n and m .

2. (2) Show by checking whether $f(t) = -f(-t)$ for odd, $f(t) = f(-t)$ for even and neither for neither which of the following are odd, even or neither: $\sin t$, $t^3 + t$, $t^3 + 2t^2$ and $|t|$.
3. (4) Find the Fourier series representation of the sawtooth function f defined by $f(t) = t$ for $-\pi < t \leq \pi$ and $f(t + 2\pi) = f(t)$.