MA22S3 Outline Solutions for Tutorial Sheet 7.[1]

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Questions

1. (2) Obtain the general solution to

$$\ddot{y} + \dot{y} - 2y = 0 \tag{1}$$

Solution: The auxiliary equation is

$$\lambda^2 + \lambda - 2 = 0 \tag{2}$$

This is factorized to give

$$(\lambda - 1)(\lambda + 2) = 0 \tag{3}$$

so the solutions are $\lambda_1 = 1$ and $\lambda_2 = -2$, so the solution is

$$y = C_1 e^t + C_2 e^{-2t} \tag{4}$$

2. (2) Obtain the general solution to

$$\ddot{y} + 6\dot{y} + 8y = 0 \tag{5}$$

Solution: The auxiliary equation is

$$\lambda^2 + 6\lambda + 8 = 0 \tag{6}$$

This is factorized to give

$$(\lambda + 4)(\lambda + 2) = 0 \tag{7}$$

so the solutions are $\lambda_1 = -4$ and $\lambda_2 = -2$, so the solution is

$$y = C_1 e^{-4t} + C_2 e^{-2t} \tag{8}$$

3. (2) Obtain the general solution to

$$2\ddot{y} + 5\dot{y} + 3y = 0 \tag{9}$$

Solution: The auxiliary equation is

$$2\lambda^2 + 5\lambda + 3 = 0 \tag{10}$$

This is factorized to give

$$(2\lambda + 3)(\lambda + 1) = 0 \tag{11}$$

so the solutions are $\lambda_1 = -3/2$ and $\lambda_2 = -1$, so the solution is

$$y = C_1 e^{-3t/2} + C_2 e^{-t} \tag{12}$$

[1]Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/MA22S3`

4. (2) Obtain the solution to

$$\ddot{y} + 7\dot{y} + 6y = 0 \tag{13}$$

with $y(0) = 2$ and $\dot{y}(2) = -1$.

Solution: The auxiliary equation is

$$\lambda^2 + 7\lambda + 6 = 0 \tag{14}$$

This is factorized to give

$$(\lambda + 1)(\lambda + 6) = 0 \tag{15}$$

so the solutions are $\lambda_1 = -1$ and $\lambda_2 = -6$, so the solution is

$$y = C_1 e^{-t} + C_2 e^{-6t} \tag{16}$$

Now $y(0) = C_1 + C_2$ and, be differentiating,

$$\dot{y} = -C_1 e^{-t} - 6C_2 e^{-6t} \tag{17}$$

and hence $\dot{y}(0) = -C_1 - 6C_2$. Thus, the initial conditions give

$$\begin{aligned} C_1 + C_2 &= 2 \\ -C_1 - 6C_2 &= -1 \end{aligned} \tag{18}$$

adding the bottom equation from the top one gives $-5C_2 = 1$ or $C_2 = -1/5$ and hence $C_1 = 11/5$. Thus

$$y = \frac{11}{5} e^{-t} - \frac{1}{5} e^{-6t} \tag{19}$$