MA22S3 Tutorial Sheet 8.[1]

3 December 2009

Useful facts:

- To solve the equation $a\ddot{y}+b\dot{y}+cy=0$, with a, b and c constants, use an exponential substitution $y=\exp(\lambda t)$ to get the auxilary equation $a\lambda^2+b\lambda+c=0$ and solve for λ. Usually this will give two solutions.
- If there is only one λ then $y=t\exp(\lambda t)$ will also be a soln.
- The solution of the equation $a\ddot{y}+b\dot{y}+cy=f(t)$ is $y=y_c+y_p$ where $y_c=C_1y_1+C_2y_2$ is the solution of $a\ddot{y}+b\dot{y}+cy=0$.
- If $f(t)=\exp(\mu t)$ and μ isn't a solution of the auxillary equation, then substitute $y=C\exp(\mu t)$ and solve for C. If μ is a solution to the auxillary equation, use $y=Ct\exp(\mu t)$ or even $y=Ct^2\exp(\mu t)$ if the auxillary equation has two repeated roots.
- If $f(t)=f_1(t)+f_2(t)$ then solve $a\ddot{y}+b\dot{y}+cy=f_1(t)$ and $a\ddot{y}+b\dot{y}+cy=f_2(t)$ and add the solutions.
- $\cosh t=[\exp(t)+\exp(-t)]/2$

Questions

1. (2) Obtain the general solution to
$$\ddot{y}+8\dot{y}+16y=0 \tag{1}$$
2. (2) Obtain the general solution to
$$\ddot{y}+16y=0 \tag{2}$$
3. (2) Obtain the solution to
$$\ddot{y}+\dot{y}-2y=e^{5t} \tag{3}$$
with $y(0)=\dot{y}(0)=0$.
4. (2) Obtain the general solution to
$$\ddot{y}+8\dot{y}+16y=4\cosh t \tag{4}$$

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