MA22S3 Tutorial Sheet 6.[1][2]

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Useful facts:

- To solve the equation $\dot{y}+py = f$ multiply across by an integrating factor $\exp\left(\int_a^t p(\tau)d\tau\right)$ and express the right hand side as the derivative of a product.
- This gives solution
$$y(t) = y(a)e^{-I(t)} + e^{-I(t)} \int_a^t f(\tau)e^{I(\tau)}d\tau \tag{1}$$
- $\int \cot t dt = \ln|\sin t| + C$

Questions

1. (2) Obtain the solution to $\dot{y} - 3y = e^{-t}$ with $y(0) = 1$; since an initial condition is chosen at $t = 0$, choose $a = 0$.
2. (3) Obtain the solution to $\dot{y} + y\cot t = \cos t$ with $y(\pi/2) = 0$.
3. (3) Obtain a general solution to $(t + 1)\dot{y} + y = (t + 1)^2$

[1]Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/MA22S3`
[2]Including material from Chris Ford, to whom many thanks.