MA22S3 Tutorial Sheet 5.[1][2]

5 November 2009

Useful facts:

- **Cosine and sine in terms of exponentials**:

$$\begin{aligned} \cos\theta &= \frac{e^{i\theta}+e^{-i\theta}}{2} \\ \sin\theta &= \frac{e^{i\theta}-e^{-i\theta}}{2i} \end{aligned} \tag{1}$$

- The **Fourier integral** or Fourier transform:

$$\begin{aligned} f(t) &= \int_{-\infty}^{\infty} dk\, \widetilde{f(k)} e^{ikt} \\ \widetilde{f(k)} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, f(t) e^{-ikt} \end{aligned}$$

- The Dirac delta function:

$$\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$$

Questions

1. (4) Express the following function as a Fourier integral:

$$f(x) = \begin{cases} \cos t & |t| < \frac{\pi}{2} \\ 0 & |t| > \frac{\pi}{2} \end{cases}$$

   One way to do the required integral is to split the cosine into exponentials.

2. (4) Do the following integrals

   (a)
$$\int_{-\infty}^{\infty} \delta(t)(t^2+3t+5)dt \tag{2}$$

   (b)
$$\int_{-\infty}^{\infty} \delta(t-\pi/4)\sin t dt \tag{3}$$

[1] Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/MA22S3`
[2] Including material from Chris Ford, to whom many thanks.

(c)

$$\int_{-\infty}^{\infty} \delta(t-1) \ln t dt \tag{4}$$

(d)

$$\int_{-3}^{3} \delta(t-4)(t^2+3t+5) dt \tag{5}$$