MA22S3 Tutorial Sheet 4.[1][2]

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Useful facts:

- **Exponential with imaginary argument**:

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{1}$$

- A function with period 2π has the **Fourier series expansion**

$$f(t) = \sum_{n=-\infty}^{\infty} c_n \exp(int).$$

where

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\exp(-int)dx$$

- **Parceval's formula**:

$$\begin{aligned}\frac{1}{l}\int_{-l/2}^{l/2} dx|f(t)|^2 &= \frac{1}{4}a_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)\\ &= \sum_{n=-\infty}^{\infty}|c_n|^2\end{aligned}$$

- The **Fourier integral** or Fourier transform:

$$\begin{aligned}f(t) &= \int_{-\infty}^{\infty} dk\,\widetilde{f(k)}e^{ikt}\\ \widetilde{f(k)} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\,f(t)e^{-ikt}\end{aligned}$$

Questions

1. (4) Express the following periodic function ($L = 2\pi$) as complex Fourier series

$$f(t) = \begin{cases} 0 & -\pi < t < -a \\ 1 & -a < t < a \\ 0 & a < t < \pi \end{cases} \tag{2}$$

where $a \in (0, \pi)$ is a constant.

2. (4) Compute the Fourier transform of $f(t) = e^{-a|t|}$ where a is a positive constant.

[1] Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/MA22S3

[2] Including material from Chris Ford, to whom many thanks.