MA22S3 Tutorial Sheet 4.¹²

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Useful facts:

• Exponential with imaginary argument:

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{1}$$

• A function with period 2π has the Fourier series expansion

$$f(t) = \sum_{n=-\infty}^{\infty} c_n \exp(int).$$

where

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) \exp\left(-int\right) dx$$

• Parceval's formula:

$$\frac{1}{l} \int_{-l/2}^{l/2} dx |f(t)|^2 = \frac{1}{4} a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} \left(a_n^2 + b_n^2 \right)$$
$$= \sum_{n=-\infty}^{\infty} |c_n|^2$$

• The Fourier integral or Fourier transform:

$$f(t) = \int_{-\infty}^{\infty} dk \, \widetilde{f(k)} e^{ikt}$$
$$\widetilde{f(k)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \, f(t) e^{-ikt}$$

Questions

1. (4) Express the following periodic function $(L = 2\pi)$ as complex Fourier series

$$f(t) = \begin{cases} 0 & -\pi < t < -a \\ 1 & -a < t < a \\ 0 & a < t < \pi \end{cases}$$
(2)

where $a \in (0, \pi)$ is a constant.

2. (4) Compute the Fourier transform of $f(t) = e^{-a|t|}$ where a is a positive constant.

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