MA22S3 Tutorial Sheet 3.[1][2]

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Useful facts:

- **Trignometric identities**: products

$$\begin{aligned}\cos A\cos B &= \frac{1}{2}[\cos(A-B)+\cos(A+B)]\\ \sin A\sin B &= \frac{1}{2}[\cos(A-B)-\cos(A+B)]\\ \sin A\cos B &= \frac{1}{2}[\sin(A+B)+\sin(A-B)].\end{aligned}\tag{1}$$

- **Trignometric identities**: double angles

$$\begin{aligned}\sin^2 A &= \frac{1}{2}(1-\cos 2A)\\ \cos^2 A &= \frac{1}{2}(1+\cos 2A)\end{aligned}\tag{2}$$

- A function $f(t)$ has period L if $f(t+L)=f(t)$, it is odd if $f(-t)=-f(t)$ and even if $f(-t)=f(t)$.

- A function with period L has the Fourier series expansion

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{2\pi nt}{L}\right)+\sum_{n=1}^{\infty}b_n\sin\left(\frac{2\pi nt}{L}\right).$$

where

$$\begin{aligned}a_0 &= \frac{2}{L}\int_{-L/2}^{L/2}f(t)dt\\ a_n &= \frac{2}{L}\int_{-L/2}^{L/2}f(t)\cos\left(\frac{2\pi nt}{L}\right)dt\\ b_n &= \frac{2}{L}\int_{-L/2}^{L/2}f(t)\sin\left(\frac{2\pi nt}{L}\right)dt\end{aligned}$$

[1]Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/MA22S3
[2]Including material from Chris Ford, to whom many thanks.

Questions

1. (4) $f(t) = \cos t$ for $-\pi/2 < t < \pi/2$ and zero for $-\pi < t < -\pi/2$ and $\pi > t > \pi/2$. It is periodic with period 2π. What is the Fourier series.

2. (2) What is the period of $\sin^2 t$ and $\sin^3 t$.

3. (2) Find the Fourier series for $\sin^3 t$; a quick way to do this is to regard it as a trigonometry problem, rather than a Fourier series problem, that is use the trigonometric identities to express it in terms of sines and cosines, rather than doing all the integrals: so start by writing $\sin^3 t = \sin^2 t \sin t$ and then write $\sin^2 t$ in terms of $\cos 2t$.