MA22S3 Tutorial Sheet 2.[1]

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Useful facts:

- Integration by parts:
$$\int_a^b u dv = uv]_a^b - \int_a^b v du \tag{1}$$
- A function $f(t)$ has period L if $f(t+L) = f(t)$, it is odd if $f(-t) = -f(t)$ and even if $f(-t) = f(t)$.
- A function with period L has the Fourier series expansion
$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n t}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n t}{L}\right).$$
where
$$\begin{aligned} a_0 &= \frac{2}{L}\int_{-L/2}^{L/2} f(t)dt \\ a_n &= \frac{2}{L}\int_{-L/2}^{L/2} f(t)\cos\left(\frac{2\pi n t}{L}\right)dt \\ b_n &= \frac{2}{L}\int_{-L/2}^{L/2} f(t)\sin\left(\frac{2\pi n t}{L}\right)dt \end{aligned}$$

Questions

1. (4) Find the Fourier series representation of the sawtooth function f defined by $f(t) = t$ for $-\pi < t \leq \pi$ and $f(t + 2\pi) = f(t)$.
2. (2) What is $f(\pi)$? If the answer to question one is
$$f(t) = -2\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\sin nt. \tag{2}$$
what value does the Fourier series give at $t = \pi$?
3. (2) By considering $t = \pi/2$ derive a formula for π.

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