## MA22S3 Tutorial Sheet 2.<sup>1</sup>

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## Useful facts:

• Integration by parts:

$$\int_{a}^{b} u dv = uv]_{a}^{b} - \int_{a}^{b} v du \tag{1}$$

- A function f(t) has period L if f(t+L) = f(t), it is odd if f(-t) = -f(t) and even if f(-t) = f(t).
- A function with period L has the Fourier series expansion

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nt}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{L}\right).$$

where

$$a_0 = \frac{2}{L} \int_{-L/2}^{L/2} f(t) dt$$
  

$$a_n = \frac{2}{L} \int_{-L/2}^{L/2} f(t) \cos\left(\frac{2\pi nt}{L}\right) dt$$
  

$$b_n = \frac{2}{L} \int_{-L/2}^{L/2} f(t) \sin\left(\frac{2\pi nt}{L}\right) dt$$

## Questions

- 1. (4) Find the Fourier series representation of the sawtooth function f defined by f(t) = t for  $-\pi < t \le \pi$  and  $f(t + 2\pi) = f(t)$ .
- 2. (2) What is  $f(\pi)$ ? If the answer to question one is

$$f(t) = -2\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin nt.$$
 (2)

what value does the Fourier series give at  $t = \pi$ ?

3. (2) By considering  $t = \pi/2$  derive a formula for  $\pi$ .

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