MA22S3 Tutorial Sheet 10[1][2]

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Useful facts:

- Series solution: assume there is a solution of the form

$$y = \sum_{n=0}^{\infty} a_n t^n$$

  and, by substituting into the equation find a recursion relation: an equation relating higher terms in a_n to lower one.

- By expanding out the sum it is easy to see $y(0) = a_0$ and $\dot{y}(0) = a_1$

- The method of Froebenius: assume there is a solution of the form

$$y = \sum_{n=0}^{\infty} a_n t^{n+s}$$

  under the assumption that a_0 is arbitrary, this assumption will give an equation for s called the indicial equation.

[1]Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/231
[2]Including material from Chris Ford, to whom many thanks.

Questions

1. (3) Use the recursion relation

$$a_{n+2} = \frac{2(n-\alpha)a_n}{(n+1)(n+2)}$$

to obtain polynomial solutions of Hermite's equation $\ddot{y} - 2t\dot{y} + 2\alpha y = 0$ for $\alpha = 3$ and 4.

2. (5) Use the method of Frobenius to obtain the general solution to the ODE

$$t\ddot{y} + 2\dot{y} + ty = 0.$$