MA22S3 Tutorial Sheet 1.[1]

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Useful facts:

- **Trignometric identities**: adding angles

$$\begin{aligned} \sin A \pm B &= \sin A \cos B \pm \cos A \sin B \\ \cos A \pm B &= \cos A \cos B \mp \sin A \sin B. \end{aligned} \tag{1}$$

- **Trignometric identities**: products

$$\begin{aligned} \cos A \cos B &= \frac{1}{2}[\cos(A-B) + \cos(A+B)] \\ \sin A \sin B &= \frac{1}{2}[\cos(A-B) - \cos(A+B)] \\ \sin A \cos B &= \frac{1}{2}[\sin(A+B) + \sin(A-B)]. \end{aligned} \tag{2}$$

- **Trignometric identities**: double angles

$$\begin{aligned} \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A. \end{aligned} \tag{3}$$

- **Even and odd**: A function is *even* if $f(t) = f(-t)$ and *odd* if $f(t) = -f(-t)$.

- **Periodic**: A function is periodic if for some constant L, $f(t+L) = f(t)$. The smallest such L is called the *period*.

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Questions

1. (2) Establish that
$$\int_{-\pi}^{\pi} dt\ \sin mt\ \cos nt = 0 \tag{4}$$
for integers n and m.

2. (2) Establish that
$$\int_{-\pi}^{\pi} dt\ \sin mt\ \sin nt = 0, \tag{5}$$
if $m \neq n$ (both m and n are positive integers). What happens when $m = n$? By the way, the same story holds for $\int_{-\pi}^{\pi} dx\ \cos mt\ \cos nt$ but you need not do that one for this problem sheet.

3. (2) Show by checking whether $f(t) = -f(-t)$ for odd, $f(t) = f(-t)$ for even and neither for neither which of the following are odd, even or neither: $\sin t$, $t^3 + t$, $t^3 + 2t^2$ and $|t|$.

4. (2) What is the period of $\sin(2\pi t/L)$ and $\cos(2\pi t/L)$?