481 Tutorial Sheet 2[1]

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Questions

These questions are all pretty straight-forward if you have the patience to read through their lengthy statement.

1. The spike triggered average is

$$\bar{A}(t) = \left\langle \sum_{t_i} s(t_i - t) \right\rangle \tag{1}$$

where $s(t)$ is the stimulus, the t_i are the spike times and the angle brackets denote the average over trials. For convenience let us write the spike triggered average of a single trial as

$$A(t) = \sum_{t_i} s(t_i - t) \tag{2}$$

so $\bar{A} = \langle A \rangle$. If we represent the spike train as

$$\rho(t) = \sum_{t_i} \delta(t - t_i) \tag{3}$$

then

$$\bar{A}(t) = \left\langle \int dt' s(t' - t)\rho(t) \right\rangle = \int dt' s(t' - t) r(t) \tag{4}$$

where the firing rate is

$$r(t) = \langle \rho(t) \rangle \tag{5}$$

If there is only a smooth number of trials, so that the rate is smoothed with a kernel

$$r(t) = \left\langle \int d\tau \rho(t - \tau) k(\tau) \right\rangle \tag{6}$$

what is $\bar{A}(t)$ in terms of $k(\tau)$ and $A(t)$? What about $Q_{\mathrm{rs}}(\tau)$ which is used in the linear rate model? These comments are intended to show that kernel smoothing, something that is always done, might be useful when presenting a rate function, but is not necessarily useful in applications of the firing rate where some other integral might provide some smoothing.

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2. For the linear rate model the integral equation for the kernel was calculated using functional differentiation, that equation was then solved by discretizing time and solving the corresponding matrix equation. Show that you get the same answer if you discretize earlier; that is, discretize the linear model

$$\tilde{r} = r_0 + \int d\tau h(\tau)s(t-\tau) \tag{7}$$

so, for example $H_i = h(i\delta t)$ and then differentiate the error with respect to H_i.

3. The convolution theorem for the Fourier transform states that

$$\mathcal{F}(f * g) = 2\pi\mathcal{F}(f)\mathcal{F}(g) \tag{8}$$

where as usual

$$\mathcal{F}(f) = \frac{1}{2\pi}\int dt f(t)e^{-ikt} \tag{9}$$

and the convolution is given by

$$f * g(t) = \int d\tau f(t)g(t-\tau) \tag{10}$$

Hence

$$\begin{aligned}
\mathcal{F}(f * g) &= \frac{1}{2\pi}\int dt \int d\tau f(t)g(t-\tau)e^{-ikt} \\
&= \int dt \int d\tau f(\tau)g(t-\tau)e^{-ik(t-\tau)}e^{-ik\tau} \\
&= \int dt \int dt' f(t)g(t')e^{-ikt'}e^{-ikt} \\
&= 2\pi\frac{1}{2\pi}\int dt f(t)e^{-ikt}\frac{1}{2\pi}\int dt g(t)e^{-ikt}
\end{aligned} \tag{11}$$

as required. You should note that in proving the convolution theorem we have assumed all the integrals run over $t \in (-\infty, \infty)$. Ignoring the finite integration limits use the convolution theorem to solve the equation for the kernel

$$\int d\tau' Q_{\mathrm{ss}}(\tau - \tau')h(\tau') = Q_{\mathrm{rs}}(-\tau). \tag{12}$$