

## 231 Tutorial Sheet 4<sup>12</sup>

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### Useful facts:

- For a scalar field  $\phi$  the gradient is

$$\text{grad } \phi = \frac{\partial \phi}{\partial x} \mathbf{i} + \frac{\partial \phi}{\partial y} \mathbf{j} + \frac{\partial \phi}{\partial z} \mathbf{k} \quad (1)$$

- For a vector field  $\mathbf{F} = (F_1, F_2, F_3)$  the divergence is

$$\text{div } \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} \quad (2)$$

- The gradient of the divergence of a scalar field is called the Laplacian:

$$\Delta = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (3)$$

- For a vector field  $\mathbf{F} = (F_1, F_2, F_3)$  the curl is

$$\text{curl } \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} \quad (4)$$

### Questions

1. Check that the Jacobian for the transformation from cartesian to spherical polar coordinates is

$$J = r^2 \sin \theta. \quad (5)$$

Consider the hemisphere defined by

$$\begin{aligned} \sqrt{x^2 + y^2 + z^2} &\leq 1 \\ z &\geq 0 \end{aligned} \quad (6)$$

Using spherical polar coordinates compute its volume and centroid.

2. Show  $\text{div } \mathbf{r} = 3$  and  $\text{grad } |\mathbf{r}| = \mathbf{r}/|\mathbf{r}|$ .
3. Find  $\nabla(1/|\mathbf{r}|)$ .
4. Show  $\text{grad } f(r) = f'(r)\hat{\mathbf{r}}$  where  $r = |\mathbf{r}|$ . If  $\mathbf{F}(r) = f(r)\mathbf{r}$  find  $\text{div } \mathbf{F}(r)$ . Find  $\text{div grad } f(r)$ .
5. Show that away from the origin the vector field

$$\mathbf{F} = \frac{\hat{\mathbf{r}}}{r^2} = \frac{\mathbf{r}}{r^3} \quad (7)$$

is irrotational (here  $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  and  $r = |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2}$ ).

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<sup>2</sup>Including material from Chris Ford, to whom many thanks.