231 Tutorial Sheet 19.[1]

24 April 2008

Useful facts:

- Laplace's equation $\triangle\phi = 0$ with Dirichlet or Neumann boundary conditions has a unique solution. This is usually proved by considering the energy like integral
$$E = \int_D dV \nabla\phi \cdot \nabla\phi \tag{1}$$

Questions

1. Prove uniqueness for solutions of the Klein-Gordon or Helmholz equation
$$\triangle\phi = m^2\phi \tag{2}$$
on a region D and with Dirichlet or Neumann boundary conditions on δD.

2. Prove uniqueness for solutions to the heat equation
$$\triangle u = k\frac{\partial u}{\partial t} \tag{3}$$
on a region $D \times [0, \infty)$ and with Dirichlet or Neumann boundary conditions on $\delta D \times [0, \infty)$, initial condition $u(\mathbf{x}, 0) = f(\mathbf{x})$ on D at time $t = 0$ and decay condition $u(\mathbf{x}, t) \to 0$ exponential fast at as t goes to infinity, k is a constant.

[1] Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/231