

## 231 Tutorial Sheet 19.<sup>1</sup>

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### Useful facts:

- Laplace's equation  $\Delta\phi = 0$  with Dirichlet or Neumann boundary conditions has a unique solution. This is usually proved by considering the energy like integral

$$E = \int_D dV \nabla\phi \cdot \nabla\phi \quad (1)$$

### Questions

1. Prove uniqueness for solutions of the Klein-Gordon or Helmholtz equation

$$\Delta\phi = m^2\phi \quad (2)$$

on a region  $D$  and with Dirichlet or Neumann boundary conditions on  $\delta D$ .

2. Prove uniqueness for solutions to the heat equation

$$\Delta u = k \frac{\partial u}{\partial t} \quad (3)$$

on a region  $D \times [0, \infty)$  and with Dirichlet or Neumann boundary conditions on  $\delta D \times [0, \infty)$ , initial condition  $u(\mathbf{x}, 0) = f(\mathbf{x})$  on  $D$  at time  $t = 0$  and decay condition  $u(\mathbf{x}, t) \rightarrow 0$  exponential fast as  $t$  goes to infinity,  $k$  is a constant.

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