231 Tutorial Sheet 16.[1,2]

6 March 2008

Useful facts:

- Euler's eqn: use $x = \exp z$.
- Series solution: assume there is a solution of the form

$$y = \sum_{n=0}^{\infty} a_n x^n$$

  and, by substituting into the equation find a recursion relation: an equation relating higher terms in a_n to lower one.

- By expanding out the sum it is easy to see $y(0) = a_0$ and $y'(0) = a_1$

Questions A short problem sheet this week!

1. Solve $x^2y'' + 4xy' + y = 0$.
2. Solve $x^2y'' + 4xy' + y = x^5$.
3. Assuming the solution of

$$(1-x)y' + y = 0 \tag{1}$$

   has a series expansion about $t = 0$ work out the recursion relation. Write out the first few terms and show that the series $a_2 = 0$ so the series actually terminates to give $y = A(1-x)$ for arbitrary A. What is the solution with $y(0) = 2$.

[1]Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/231
[2]Including material from Chris Ford, to whom many thanks.