231 Tutorial Sheet 14.[1,2]

2 Febuary 2008

Useful facts:

- The Dirac delta function:
$$\int_{-\infty}^{\infty} dx\, f(x)\delta(x) = f(0)$$
- If $h(x)$ is a continuous function
$$\int_{-\infty}^{\infty} dx\, f(x)\delta[h(x)] = \sum_{x_i : f(x_i)=0} \frac{f(x_i)}{|h'(x_i)|}$$
- To solve the equation $y' + py = f$ multiply across by an integrating factor $\exp\left(\int^t d\tau p(\tau)\right)$ and express the right hand side as the derivative of a product.
- To solve the equation $ay'' + by' + cy = 0$, with a, b and c constants, use an exponential substitution $y = \exp(\lambda x)$ and solve for λ. If this only gives one solution, then $y = x\exp(\lambda x)$ is also a solution.

[1]Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/231
[2]Including material from Chris Ford, to whom many thanks.

Questions

1. Inside an integral, what is
$$\frac{d}{dx}\frac{1}{1+\epsilon\theta(x)} \tag{1}$$
for $\theta(x)$ the usual Heaviside function and $|\epsilon| < 1$.

2. Compute
   (a) $\int_{-\infty}^{\infty} dx\ e^x\ \delta(x+1)$
   (b) $\int_{-3}^{1} dx\ \delta(x^2 - 3x + 2)$
   (c) $\int_{-\infty}^{\infty} dx\ \ \cos x\ \delta'(x)$
   (d) $\int_0^1 dx\ \delta\left(\sin\frac{1}{x}\right)$.

3. Obtain a general solution to
   (a) $y' - 3y = e^{-x}$
   (b) $y' + y\cot x = \cos x$
   (c) $(x+1)y' + y = (x+1)^2$

4. Obtain the general solutions of the following ODEs:
   (a) $y'' + 5y' + 6y = 0$
   (b) $y'' - 2y' + y = 0$