

231 Tutorial Sheet 14.¹²

2 February 2008

Useful facts:

- The Dirac delta function:

$$\int_{-\infty}^{\infty} dx f(x) \delta(x) = f(0)$$

- If $h(x)$ is a continuous function

$$\int_{-\infty}^{\infty} dx f(x) \delta[h(x)] = \sum_{x_i: f(x_i)=0} \frac{f(x_i)}{|h'(x_i)|}$$

- To solve the equation $y' + py = f$ multiply across by an integrating factor $\exp(\int^t d\tau p(\tau))$ and express the right hand side as the derivative of a product.
- To solve the equation $ay'' + by' + cy = 0$, with a , b and c constants, use an exponential substitution $y = \exp(\lambda x)$ and solve for λ . If this only gives one solution, then $y = x \exp(\lambda x)$ is also a solution.

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²Including material from Chris Ford, to whom many thanks.

Questions

1. Inside an integral, what is

$$\frac{d}{dx} \frac{1}{1 + \epsilon \theta(x)} \quad (1)$$

for $\theta(x)$ the usual Heaviside function and $|\epsilon| < 1$.

2. Compute

(a) $\int_{-\infty}^{\infty} dx \, e^x \, \delta(x+1)$

(b) $\int_{-3}^1 dx \, \delta(x^2 - 3x + 2)$

(c) $\int_{-\infty}^{\infty} dx \, \cos x \, \delta'(x)$

(d) $\int_0^1 dx \, \delta\left(\sin \frac{1}{x}\right)$.

3. Obtain a general solution to

(a) $y' - 3y = e^{-x}$

(b) $y' + y \cot x = \cos x$

(c) $(x+1)y' + y = (x+1)^2$

4. Obtain the general solutions of the following ODEs:

(a) $y'' + 5y' + 6y = 0$

(b) $y'' - 2y' + y = 0$