

231 Tutorial Sheet 14.¹²

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Useful facts:

- The Dirac delta function:

$$\int_{-\infty}^{\infty} dx f(x) \delta(x) = f(0)$$

- If $h(x)$ is a continuous function

$$\int_{-\infty}^{\infty} dx f(x) \delta[h(x)] = \sum_{x_i: f(x_i)=0} \frac{f(x_i)}{|h'(x_i)|}$$

- To solve the equation $y' + py = f$ multiply across by an integrating factor $\exp(\int^t d\tau p(\tau))$ and express the right hand side as the derivative of a product.
- To solve the equation $ay'' + by' + cy = 0$, with a , b and c constants, use an exponential substitution $y = \exp(\lambda x)$ and solve for λ . If this only gives one solution, then $y = x \exp(\lambda x)$ is also a solution.

Questions

1. Inside an integral, what is

$$\frac{d}{dx} \frac{1}{1 + \epsilon \theta(x)} \quad (1)$$

for $\theta(x)$ the usual Heaviside function and $|\epsilon| < 1$.

2. Compute

$$(a) \int_{-\infty}^{\infty} dx e^x \delta(x+1)$$

$$(b) \int_{-3}^1 dx \delta(x^2 - 3x + 2)$$

$$(c) \int_{-\infty}^{\infty} dx \cos x \delta'(x)$$

$$(d) \int_0^1 dx \delta\left(\sin \frac{1}{x}\right).$$

3. Obtain a general solution to

$$(a) y' - 3y = e^{-x}$$

$$(b) y' + y \cot x = \cos x$$

$$(c) (x+1)y' + y = (x+1)^2$$

4. Obtain the general solutions of the following ODEs:

$$(a) y'' + 5y' + 6y = 0$$

$$(b) y'' - 2y' + y = 0$$

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²Including material from Chris Ford, to whom many thanks.