

## 231 Tutorial Sheet 8<sup>12</sup>

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### Useful facts:

- To evaluate the line integral for a parameterized curve:

$$\int_c \mathbf{F} \cdot d\mathbf{l} = \int_{t_1}^{t_2} \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} dt \quad (1)$$

where  $t_1$  and  $t_2$  are the parameter values corresponding to the beginning and end of the curve.

- To evaluate the surface integral for a parameterized surface:

$$\int \int_S \mathbf{F} \cdot d\mathbf{A} = \int \int_S \mathbf{F} \cdot \left( \frac{d\mathbf{r}}{du} \times \frac{d\mathbf{r}}{dv} \right) du dv \quad (2)$$

- Stokes' theorem: for an orientable piecewise smooth surface  $S$  with an orientable piecewise smooth boundary  $C$  oriented so that  $\mathbf{N} \times d\mathbf{l}$  points into  $S$  with  $\mathbf{n}$  the normal and  $d\mathbf{l}$  a tangent to  $C$  and  $\mathbf{F}$  a vector field defined in a region containing  $S$ , then

$$\int_S \text{curl } \mathbf{F} \cdot d\mathbf{A} = \int_C \mathbf{F} \cdot d\mathbf{l} \quad (3)$$

- If  $S$  is a surface  $\partial S$  is its bounding curve with the orientation describe above.
- Green's theorem on the plane: let  $D$  be a region in the  $xy$ -plane bounded by a piecewise continuous curve  $C$ , if  $f(x, y)$  and  $g(x, y)$  have continuous first derivatives

$$\int_D \left( \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dx dy = \int_C (f dx + g dy) \quad (4)$$

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## Questions

1. Find the flux of  $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + \mathbf{k}$  across the portion of the paraboloid

$$\mathbf{r}(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + (1 - u^2) \mathbf{k} \quad (5)$$

with  $1 \leq u \leq 2$  and  $0 \leq v \leq 2\pi$ , oriented to give a positive answer.

2. Find the flux of  $\mathbf{F} = e^{-y}\mathbf{i} - y\mathbf{j} + x \sin z \mathbf{k}$  across the portion of the paraboloid

$$\mathbf{r}(u, v) = 2 \cos v \mathbf{i} + \sin v \mathbf{j} + u \mathbf{k} \quad (6)$$

with  $0 \leq u \leq 5$  and  $0 \leq v \leq 2\pi$ , oriented to give a positive answer.

3. Use Green's Theorem to evaluate

$$\oint_C (y^2 dx + x^2 dy) \quad (7)$$

where  $C$  is the square with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(1, 1)$  and  $(0, 1)$  and oriented anti-clockwise.

4. Calculate directly and using Stoke's Theorem

$$\int_S \mathbf{F} \cdot d\mathbf{S} \quad (8)$$

where  $\mathbf{F} = (z - y)\mathbf{i} + (z + x)\mathbf{j} - (x + y)\mathbf{k}$  and  $S$  is the paraboloid  $z = 9 - x^2 - y^2$  oriented upwards.