231 Tutorial Sheet 8[1][2]

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Useful facts:

- To evaluate the line integral for a parameterized curve:

$$\int_c \mathbf{F}\cdot\mathbf{dl} = \int_{t_1}^{t_2} \mathbf{F}\cdot\frac{d\mathbf{r}}{dt}dt \tag{1}$$

  where t_1 and t_2 are the parameter values corresponding to the begining and end of the curve.

- To evaluate the surface integral for a parameterized surface:

$$\int\int_S \mathbf{F}\cdot\mathbf{dA} = \int\int_S \mathbf{F}\cdot\left(\frac{d\mathbf{r}}{du}\times\frac{d\mathbf{r}}{dv}\right)dudv \tag{2}$$

- Stokes' theorem: for an orientable piecewise smooth surface S with an orientable piecewise smooth boundy C oriented so that $\mathbf{N}\times\mathbf{dl}$ points into S with $\mathbf{n}$ the normal and $\mathbf{dl}$ a tangent to C and $\mathbf{F}$ a vector field defined in a region containing S, then

$$\int_S \text{curl}\,\mathbf{F}\cdot\mathbf{dA} = \int_C \mathbf{F}\cdot\mathbf{dl} \tag{3}$$

- If S is a surface δS is its bounding curve with the orientation describe above.

- Green's theorem on the plane: led D be a region in the xy-plane bounded by a piecewise continuous curve C, if $f(x,y)$ and $g(x,y)$ have continuous first derivatives

$$\int_D\left(\frac{\partial g}{\partial x}-\frac{\partial f}{\partial y}\right)dxdy = \int_C (fdx+gdy) \tag{4}$$

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[2]Including material from Chris Ford, to whom many thanks.

Questions

1. Find the flux of $\mathbf{F}=x\mathbf{i}+y\mathbf{j}+\mathbf{k}$ across the portion of the paraboloid

$$\mathbf{r}(u,v)=u\cos v\mathbf{i}+u\sin v\mathbf{j}+(1-u^2)\mathbf{k} \quad (5$$

   with $1\le u\le 2$ and $0\le v\le 2\pi$, oriented to give a positive answer.

2. Find the flux of $\mathbf{F}=e^{-y}\mathbf{i}-y\mathbf{j}+x\sin z\mathbf{k}$ across the portion of the paraboloid

$$\mathbf{r}(u,v)=2\cos v\mathbf{i}+\sin v\mathbf{j}+u\mathbf{k} \quad (6$$

   with $0\le u\le 5$ and $0\le v\le 2\pi$, oriented to give a positive answer.

3. Use Green's Theorem to evaluate

$$\oint_c (y^2dx+x^2dy) \quad (7$$

   where C is the square with vertice $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$ and oriented ant[cut off] clockwise.

4. Calculate directly and using Stoke's Theorem

$$\int_S \mathbf{F}\cdot\mathbf{dS} \quad (8$$

   where $\mathbf{F}=(z-y)\mathbf{i}+(z+x)\mathbf{j}-(x+y)\mathbf{k}$ and S is the paraboloid $z=9-x^2-y$[cut off] oriented upwards.