231 Tutorial Sheet 4[1,2]

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Useful facts:

- For a scalar field ϕ the gradiant is

$$\text{grad}\,\phi = \frac{\partial\phi}{\partial x}\mathbf{i} + \frac{\partial\phi}{\partial y}\mathbf{j} + \frac{\partial\phi}{\partial z}\mathbf{k} \tag{1}$$

- For a vector field $\mathbf{F} = (F_1, F_2, F_3)$ the divergence is

$$\text{div}\,\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} \tag{2}$$

Questions

1. Show that away from the origin the vector field

$$\mathbf{F} = \frac{\hat{\mathbf{r}}}{r^2} = \frac{\mathbf{r}}{r^3}$$

   is divergenceless.

2. Show $\text{div}\,\mathbf{r} = 3$ and $\text{grad}\,|\mathbf{r}| = \mathbf{r}/|\mathbf{r}|$.

3. Find $\nabla(1/|\mathbf{r}|)$.

4. Show $\text{grad}\,f(r) = f'(r)\hat{\mathbf{r}}$ where $r = |\mathbf{r}|$. If $\mathbf{F}(r) = f(r)\mathbf{r}$ find $\text{div}\mathbf{F}(r)$. Find $\text{div}\,\text{grad}\,f(r)$.

Useful fact:

- The gradient of the divergence of a scalar field is called the Laplacian:

$$\triangle = \nabla\cdot\nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{3}$$

  and is an important operator.

[1]Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/231
[2]Including material from Chris Ford, to whom many thanks.