231 Tutorial Sheet 21, due Friday 11 May[1]

4 May 2007

Useful facts:

- **Seperation of variables**: for example, let $\phi(x,y) = X(x)Y(y)$ and substitute into the equation.

Questions

1. The function $\phi(x,y)$ is harmonic in the square $0 \le x \le \pi$, $0 \le y \le \pi$. On three sides ϕ is zero and on the lower side
$$\phi(x,0) = \cos x. \tag{1}$$
Determine $\phi(x,y)$ within the square. Sketch the lines of constant ϕ.

2. Repeat the problem with the boundary condition $\phi(x,0) = \sin x$ (again ϕ is zero on the other three sides).

3. Repeat the problem with the Neumann boundary condition
$$\frac{\partial \phi}{\partial x} = 0 \tag{2}$$
at $x = 0$ and $x = \pi$,
$$\frac{\partial \phi}{\partial y} = 0 \tag{3}$$
at $y = \pi$ and
$$\frac{\partial \phi}{\partial y} = -1 \tag{4}$$
on $y = 0$

4. $\phi(x,y)$ is harmonic in the strip $0 \le y \le 1$ and periodic in the x direction. On the upper and lower edges Dirichlet boundary conditions are imposed
$$\phi(x, y = 1) = 1 + \sin x, \qquad \phi(x, y = 0) = \cos 2x.$$
Determine $\phi(x,y)$ within the strip.

   Suggestion: When applying the separation of variables method do not forget the case
$$\frac{X''(x)}{X(x)} = \frac{Y''(y)}{Y(y)} = 0. \tag{5}$$

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