

231 Tutorial Sheet 21, due Friday 11 May¹

4 May 2007

Useful facts:

- **Seperation of variables:** for example, let $\phi(x, y) = X(x)Y(y)$ and substitute into the equation.

Questions

1. The function $\phi(x, y)$ is harmonic in the square $0 \leq x \leq \pi$, $0 \leq y \leq \pi$. On three sides ϕ is zero and on the lower side

$$\phi(x, 0) = \cos x. \quad (1)$$

Determine $\phi(x, y)$ within the square. Sketch the lines of constant ϕ .

2. Repeat the problem with the boundary condition $\phi(x, 0) = \sin x$ (again ϕ is zero on the other three sides).
3. Repeat the problem with the Neumann boundary condition

$$\frac{\partial \phi}{\partial x} = 0 \quad (2)$$

at $x = 0$ and $x = \pi$,

$$\frac{\partial \phi}{\partial y} = 0 \quad (3)$$

at $y = \pi$ and

$$\frac{\partial \phi}{\partial y} = -1 \quad (4)$$

on $y = 0$

4. $\phi(x, y)$ is harmonic in the strip $0 \leq y \leq 1$ and periodic in the x direction. On the upper and lower edges Dirichlet boundary conditions are imposed

$$\phi(x, y = 1) = 1 + \sin x, \quad \phi(x, y = 0) = \cos 2x.$$

Determine $\phi(x, y)$ within the strip.

Suggestion: When applying the separation of variables method do not forget the case

$$\frac{X''(x)}{X(x)} = \frac{Y''(y)}{Y(y)} = 0. \quad (5)$$

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