231 Tutorial Sheet 20.¹

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Useful facts:

• Laplace's equation $\Delta \phi = 0$ with Dirichlet or Neumann boundary conditions has a unique solution. This is usually proved by considering the energy like integral

$$E = \int_{D} dV \nabla \phi \cdot \nabla \phi \tag{1}$$

Questions

1. Prove uniqueness for solutions of the Klein-Gordon or Helmholz equation

$$\Delta \phi = m^2 \phi \tag{2}$$

on a region D and with Dirichlet or Neumann boundary conditions on δD .

2. Prove uniqueness for solutions to the heat equation

$$\Delta u = k \frac{\partial u}{\partial t} \tag{3}$$

on a region $D \times [0, \infty)$ and with Dirichlet or Neumann boundary conditions on $\delta D \times [0, \infty)$, initial condition $u(\mathbf{x}, 0) = f(\mathbf{x})$ on D at time t = 0 and decay condition $u(\mathbf{x}, t) \to 0$ exponential fast at as t goes to infinity, k is a constant.

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