**231 Tutorial Sheet 2**[1][2]

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**Useful facts:**

- The *iterated integral* is the integral expressed as a series of nested one-dimensional integrals.
- The two-dimensional area element $dA = dxdy = rdrd\theta$

**Questions**

1. Consider the integral
$$I = \int_D dV\ \phi \tag{1}$$
where $D$ is the interior of the ellipsoid defined by
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \tag{2}$$
Write down $I$ as an iterated triple integral.

2. The Gaussian integral formula
$$\int_{-\infty}^{\infty} dx\ e^{-x^2} = \sqrt{\pi} \tag{3}$$
can be derived easily with the help of polar coordinates. The trick is to note that the *square* of the integral can be recast as a double integral over $R^2$:
$$\left(\int_{-\infty}^{\infty} dx\ e^{-x^2}\right)^2 = \int_{R^2} dA\ e^{-x^2-y^2}. \tag{4}$$
By changing to polar coordinates evaluate this integral.

3. Compute the Jacobian of the transformation from cartesian to parabolic cylinder coordinates
$$x = \frac{1}{2}(u^2 - v^2), \qquad y = uv.$$

4. Determine the volume of the region enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. *Suggestion: Use Cartesian coordinates.*

[1]Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/231`

[2]Including material from Chris Ford, to whom many thanks.