

231 Tutorial Sheet 2¹²

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Useful facts:

- The *iterated integral* is the integral expressed as a series of nested one-dimensional integrals.
- The two-dimensional area element $dA = dxdy = r dr d\theta$

Questions

1. Consider the integral

$$I = \int_D dV \phi \quad (1)$$

where D is the interior of the ellipsoid defined by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \quad (2)$$

Write down I as an iterated triple integral.

2. The Gaussian integral formula

$$\int_{-\infty}^{\infty} dx e^{-x^2} = \sqrt{\pi} \quad (3)$$

can be derived easily with the help of polar coordinates. The trick is to note that the *square* of the integral can be recast as a double integral over R^2 :

$$\left(\int_{-\infty}^{\infty} dx e^{-x^2} \right)^2 = \int_{R^2} dA e^{-x^2 - y^2}. \quad (4)$$

By changing to polar coordinates evaluate this integral.

3. Compute the Jacobian of the transformation from cartesian to parabolic cylinder coordinates

$$x = \frac{1}{2}(u^2 - v^2), \quad y = uv.$$

4. Determine the volume of the region enclosed by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. *Suggestion: Use Cartesian coordinates.*

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²Including material from Chris Ford, to whom many thanks.