231 Tutorial Sheet 8[1,2]

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Useful facts:

- The vector potential: on a domain with no internal boundary, a smooth vector vector field $\mathbf{F}$ with zero divergence has a vector field $\mathbf{A}$ so that $\mathbf{F} = \nabla \times \mathbf{A}$.

- Hodge decomposition: under some conditions, a general smooth vector field can be written in the form $\mathbf{F} = \nabla \times \mathbf{A} + \nabla\phi$ for some smooth vector field $\mathbf{A}$ and some scalar field ϕ.

Questions

1. Obtain a vector potential for the solenoidal vector field: $\mathbf{F} = x\mathbf{i} + y\mathbf{j} - 2z\mathbf{k}$

2. Obtain a vector potential for the solenoidal vector field: $\mathbf{F} = e^x\mathbf{k}$.

3. Find a Hodge decomposition for the vector field $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$.

[1]Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/231`
[2]Including material from Chris Ford, to whom many thanks.