231 Tutorial Sheet 5[1][2]

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Useful facts:

- To evaluate the line integral for a parameterized curve:

$$\int_c \mathbf{F}\cdot\mathbf{dl} = \int_{t_1}^{t_2} \mathbf{F}\cdot\frac{d\mathbf{r}}{dt}dt \tag{1}$$

  where t_1 and t_2 are the parameter values corresponding to the begining and end of the curve.

- To evaluate the surface integral for a parameterized surface:

$$\int\int_S \mathbf{F}\cdot\mathbf{dA} = \int\int_S \mathbf{F}\cdot\left(\frac{d\mathbf{r}}{du}\times\frac{d\mathbf{r}}{dv}\right)dudv \tag{2}$$

Questions

1. (a) Compute the flux of the vector field $\mathbf{F} = x\mathbf{i}+2y\mathbf{j}+z\mathbf{k}$ out of the cylinder defined by $x^2+y^2=1$ and $0\le z\le 1$.
   (b) What is the flux if the vector field is replaced with $\mathbf{F}=\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$?
   (c) Find the flux of $\mathbf{F}=z^2\mathbf{k}$ upwards through the part of the sphere $x^2+y^2+z^2=a^2$ in the first octant of $\mathbf{R}^3$ (in the first octant all three coordinates x, y and z are positive).
2. Find the flux of $\mathbf{F}=2x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ upwards through the surface with parametrization $\mathbf{r}(u,v)=u^2v\mathbf{i}+uv^2\mathbf{j}+v^3\mathbf{k}$ where u and v range from 0 to 1.

[1] Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/231
[2] Including material from Chris Ford, to whom many thanks.