231 Tutorial Sheet 1[1][2]

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Useful facts:

- The *iterated integral* is the integral expressed as a series of nested one-dimensional integrals.
- The two-dimensional area element $dA = dxdy = rdrd\theta$

Questions

1. Rewrite the integral
$$I = \int_0^1 dx \int_1^{e^x} dy\ \phi(x,y) \tag{1}$$
as a double integral with the opposite order of integration.

2. Consider the integral
$$I = \int_D dV\ \phi \tag{2}$$
where D is the interior of the ellipsoid defined by
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1. \tag{3}$$
Write down I as an iterated triple integral.

3. The Gaussian integral formula
$$\int_{-\infty}^{\infty} dx\ e^{-x^2} = \sqrt{\pi} \tag{4}$$
can be derived easily with the help of polar coordinates. The trick is to note that the *square* of the integral can be recast as a double integral over R^2:
$$\left(\int_{-\infty}^{\infty} dx\ e^{-x^2}\right)^2 = \int_{R^2} dA\ e^{-x^2-y^2}. \tag{5}$$
By changing to polar coordinates evaluate this integral.

[1]Conor Houghton, houghton@maths.tcd.ie, see also http://www.maths.tcd.ie/~houghton/231

[2]Including material from Chris Ford, to whom many thanks.