2007 Schol exam, q6 outline solutions.

Write the triangular pulse

$$f(t) = \begin{cases} At/T + A & -T < t < 0 \\ -At/T + A & 0 < t < T \\ 0 & |t| > T \end{cases}$$

as a Fourier integral.

*Solution:*So, substituting in

$$\begin{aligned} \tilde{f(k)} &= \frac{1}{2\pi}\int_{\infty}^{\infty} f(t)e^{-ikt}dt \\ &= \frac{1}{2\pi}\int_{-T}^{0}\left(\frac{At}{T}+A\right)e^{-ikt}dt + \frac{1}{2\pi}\int_{0}^{T}\left(-\frac{At}{T}+A\right)e^{-ikt}dt \end{aligned} \tag{1}$$

Now, doing a change of variable from t to $-t$ gives

$$\int_{-T}^{0}\frac{At}{T}e^{-ikt}dt = -\int_{0}^{T}\frac{At}{T}e^{ikt}dt \tag{2}$$

so we can put everything together

$$\tilde{f(k)} = \frac{A}{2\pi}\int_{-T}^{T}e^{-ikt}dt - \frac{A}{\pi T}\int_{0}^{T}t\cos ktdt \tag{3}$$

The second integral has to be done by parts

$$\begin{aligned} \int_{0}^{T}t\cos ktdt &= \frac{1}{k}t\sin kt\Big]_{0}^{T} - \frac{1}{k}\int_{0}^{T}\sin ktdt \\ &= \frac{T}{k}\sin kT + \frac{1}{k^2}\cos kT - \frac{1}{k^2} \end{aligned} \tag{4}$$

The first integral is

$$\int_{-T}^{T}e^{-ikt} = \frac{1}{-ik}e^{-ikt}\Big]_{-T}^{T} = \frac{2}{k}\sin kT \tag{5}$$

Thus, we have

$$\tilde{f(k)} = \frac{A}{\pi k}\sin kT - \frac{A}{\pi T}\left(\frac{T}{k}\sin kT + \frac{1}{k^2}\cos kT - \frac{1}{k^2}\right) = \frac{A}{\pi T k^2}(1-\cos kT) \tag{6}$$