

14 May
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$$\text{curl } \underline{F} = \begin{pmatrix} \partial_y F_3 - \partial_z F_2 \\ \partial_z F_1 - \partial_x F_3 \\ \partial_x F_2 - \partial_y F_1 \end{pmatrix}$$

$$\begin{aligned} (\text{curl curl } \underline{F})_1 &= \partial_y (\partial_x F_2 - \partial_y F_1) \\ &\quad - \partial_z (\partial_z F_1 - \partial_x F_3) \\ &= \partial_y \partial_x F_2 + \partial_z \partial_x F_3 + \underbrace{\partial_x \partial_x F_1}_{\sim} \\ &\quad - \underbrace{(\partial_x^2 + \partial_y^2 + \partial_z^2)}_{\sim} F_1 \\ &= \partial_x (\underline{\nabla} \cdot \underline{F}) - \Delta F_1 \\ &= (\text{grad div } \underline{F} - \Delta \underline{F})_1 \end{aligned}$$

$$\underline{H} = \text{curl} \frac{\partial \underline{Z}}{\partial t}$$

$$\underline{E} = \text{curl} \text{curl} \underline{Z}$$

$$\text{div} \underline{H} = \text{div} \left(\text{curl} \frac{\partial \underline{Z}}{\partial t} \right) = 0$$

$$\therefore \text{div} \text{curl} \underline{F} = 0$$

$\sqrt{\underline{F}}$

$$\text{div} \underline{E} = \text{div} \left(\text{curl} (\text{curl} \underline{Z}) \right)$$

$$= 0 \quad \text{as above}$$

$$\text{curl} \underline{H} = \text{curl} \text{curl} \frac{\partial \underline{Z}}{\partial t}$$

$$= \frac{\partial}{\partial t} \text{curl} \text{curl} \underline{Z} = \frac{\partial \underline{E}}{\partial t}$$

$$\text{curl} \underline{E} = \text{curl} \left(\text{grad} \text{div} \underline{Z} - \Delta \underline{Z} \right)$$

$$= - \text{curl} \frac{\partial^2 \underline{Z}}{\partial t^2} = - \frac{\partial \underline{H}}{\partial t} \quad \because \text{curl} \text{grad} \phi = 0$$

$$\& \text{ using } \Delta \underline{Z} = \frac{\partial^2 \underline{Z}}{\partial t^2}$$