231 Annual Exam Outline Soln 2015

q4

$$\text{curl curl } \underline{F} = \text{grad div } F - \Delta F.$$

$$\text{curl } \underline{F} = \begin{pmatrix} \partial_y F_3 - \partial_z F_2 \\ \partial_z F_1 - \partial_x F_3 \\ \partial_x F_2 - \partial_y F_1 \end{pmatrix}$$

$$\left(\text{curl curl } \underline{F}\right)_1 = \partial_y(\partial_1 F_2 - \partial_y F_1) - \partial_z(\partial_z F_1 - \partial_1 F_3)$$

(2) $$= \partial_y \partial_1 F_2 - \partial_y \partial_y F_1 - \partial_z \partial_z F_1 + \partial_z \partial_1 F_3$$

(1)

$$\text{div } \underline{F} = \partial_x F_1 + \partial_y F_2 + \partial_z F_3$$

$$\left(\text{grad div } \underline{F}\right)_1 = \partial_1^2 F_1 + \partial_1 \partial_y F_2 + \partial_1 \partial_z F_3 \quad (1)$$

$$\left(\Delta \underline{F}\right)_1 = \partial_1^2 F_1 + \partial_y^2 F_1 + \partial_z^2 F_1 \quad (1)$$

as required

(4) $\operatorname{div} H = \underbrace{\operatorname{div}\operatorname{curl}}\ \frac{\partial Z}{\partial t} = 0$

(4) $\operatorname{div} E = \underbrace{\operatorname{div}\operatorname{curl}}\ \operatorname{curl}\ Z = 0$

(3) $\operatorname{curl} H = \operatorname{curl}\operatorname{curl}\left(\frac{\partial Z}{\partial t}\right) = \operatorname{grad}\operatorname{div}\frac{\partial Z}{\partial t} - \Delta\frac{\partial Z}{\partial t}$

$= \frac{\partial E}{\partial t} = \operatorname{curl}\operatorname{curl}\frac{\partial Z}{\partial t}$

(4) $\operatorname{curl} E + \frac{\partial H}{\partial t} = \operatorname{curl}\left(\frac{\partial^2 Z}{\partial t^2} - \Delta Z\right) = 0$