

2007

231 Annual Exam Outline Solns

14
June 07

(a) What is a Jacobian

4

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos & -r \sin \\ \sin & r \cos \end{vmatrix}$$

$$= r$$

4

(b)

$$A = \iint dA = \int_0^{2\pi} d\theta \int_0^{\sin 3\theta} dr r$$

$$= \int_0^{2\pi} d\theta \int dr \frac{1}{2} r^2$$

$$= \int_0^{2\pi} d\theta \frac{1}{2} \sin^2 3\theta$$

$$= \frac{1}{2} \int_0^{2\pi} d\theta \frac{1}{2} - \text{periodic stuff.}$$

$$= \frac{\pi}{2}$$

4

$$\begin{aligned} r^2 + c^2 &= 1 \\ c^2 - r^2 &= \cos 2\theta \\ r^2 &= \frac{1 - \cos 2\theta}{2} \end{aligned}$$

4