28 May 07

231 Summer Exam <u>OUTLINE</u> SOLN

q5

$$8x^2y'' + 10xy' + (x-1)y = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^{n+r}$$

$$xy' = \sum_{n=0}^{\infty} (n+r)\, a_n x^{n+r}$$

$$x^2y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1)\, a_n x^{n+r}$$

however $xy = \sum_{n=0}^{\infty} a_n x^{n+r+1}$

so we need to move everything up one

$$xy' = \sum_{n=0}^{\infty} (n+r) a_n x^{n+r} = \sum_{m=-1}^{\infty} (m+r+1) a_{m+1} x^{m+r+1}$$

$$= r a_0 x^r + \sum_{n=0}^{\infty} (n+r+1) a_{n+1} x^{n+r+1}$$

$$x^2y'' = \sum_{n=0}^{\infty} (n+r)(n+r-1) a_n x^{n+r} = \sum_{m=-1}^{\infty} (m+r+1)(m+r) a_{m+1} x^{m+r+1}$$

$$= r(r-1) a_0 x^r + \sum_{n=0}^{\infty} (n+r+1)(n+r) a_{n+1} x^{n+r+1}$$

$$y = \sum_{n=0}^{\infty} a_n x^{n+r} = a_0 x^r + \sum_{n=0}^{\infty} a_{n+1} x^{n+r+1}$$

$$8x^2y'' + 10xy' + (x-1)y = 0$$

giving

$$10 r a_0 x^r + 8r(r-1)a_0 x^r - a_0 x^r$$

$$+ \sum_{n=0}^{\infty} \left\{ \left[8(m+r)(m+r-1) + 10(m+r) - 1 \right] a_{n+1} + a_n \right\} x^{n+r+1} = 0$$

hence

$$(8r^2 - 8r + 10r - 1) a_0 = 0$$

so the indicial eqn is

$$8r^2 + 2r - 1 = 0$$

$$r = \frac{-2 \pm \sqrt{4+32}}{16}$$

$$= -\frac{1}{8} \pm \frac{1}{\cancel{16}_{8}} \times \cancel{6}^{3}$$

$$= -\frac{1}{8} \pm \frac{3}{8}$$

so for a soln which is nonsingular at the origin

$$r = \frac{1}{4}$$

recursion relation $a_{n+1} = \dfrac{-a_n}{8(n+\frac{1}{4})(n-\frac{3}{4})+10(n+\frac{1}{4})-1}$

$$8(n^2 - \tfrac{1}{2}n - \tfrac{3}{16}) + 10n + \frac{5}{2} - 1$$
$$= 8n^2 - 4n - \frac{3}{2} + 10n + \frac{5}{2} - 1$$
$$= 8n^2 + 6n$$

$y = x^{\frac{1}{4}} \sum_{n=0}^{\infty} a_n x^n \qquad a_{n+1} = \dfrac{-a_n}{8n^2+6n}$

$$y(1) = a_0 + a_1 + a_2 + \ldots$$
$= a_0\left(1 - \dfrac{1}{14}\right) = 1 \qquad a_0 = \dfrac{14}{13}$

≈ 1.07

next term $\dfrac{1}{14} \dfrac{1}{32+12} = \dfrac{1}{14} \times \dfrac{1}{44}$

so 1.07 is accurate enough!