231 Annual Exam OUTLINE SOLN

q5 (a) $y'' + 4y' + 4y = e^{-2x}\cos 2x$

check $(\lambda^2 + 4\lambda + 4) = 0$

$$\lambda = \frac{-4 \pm \sqrt{16-16}}{2}$$

$$= -2$$

$$e^{-2x}\cos 2x = e^{-2x}\left(\frac{e^{2ix} + e^{-2ix}}{2}\right)$$

$$= \frac{1}{2}e^{2(-1+i)x} + \frac{1}{2}e^{2(-1-i)x}$$

$$y = Ce^{2(-1+i)x}$$

$$y' = 2(-1+i)Ce^{2(-1+i)x}$$

$$y'' = 4(-1+i)^2 Ce^{2(-1+i)x}$$

$$(-1+i)^2 = 1 - 2i - 1 = -2i$$

$\Rightarrow$

$$-8iC + 4(-1+i)C + 4C = \frac{1}{2}$$

$$-4iC = \frac{1}{2} \qquad C = \frac{i}{8}$$

$\Rightarrow$ particular sol^n is

$$y = -\frac{e^{-2x}}{4}\left(\frac{e^{2ix} - e^{-2ix}}{2i}\right)$$

$$= -\frac{1}{4} e^{-2x} \sin 2x$$

(b) $y(x) = 1$

$y' = y'' = 0$

$4x1 = 4$ so is a sol^n.

$$y'' + 4y + 4 = 0$$

$$\lambda^2 + 4\lambda + 4 = 0$$

$$(\lambda + 2)(\lambda + 2) = 0$$

$\Rightarrow$ homogeneous sol^n is

$$y = C_1 e^{-2x} + C_2 x e^{-2x}$$

& The sol'n to $y'' + 4y' + 4y = 0$ is

$$y = C_1 e^{-2x} + C_2 x e^{-2x} + 1$$

$$y(0) = C_1 + 1 \qquad C_1 = -1$$

$$y'(x) = -2C_1 e^{-2x} + C_2 e^{-2x} - 2C_2 x e^{-2x}$$

$$y'(0) = C_2 - 2C_1$$

$$\Rightarrow \quad C_2 = -2$$

& $$y(x) = -e^{-2x} - 2x e^{-2x} + 1$$