231 Annual Exam 2006 <u>OUTLINE SOLNS</u>

May 07

q2 first part from notes

$$I = \int \underline{F} \cdot d\underline{r} = \int_C (2x^2 dx + 2yz\,dy + y^2 dz)$$

$$\underline{F} = 2x^2 \underline{i} + 2yz \underline{j} + y^2 \underline{k}$$
$$= \frac{\partial \phi}{\partial x} \underline{i} + \frac{\partial \phi}{\partial y} \underline{j} + \frac{\partial \phi}{\partial z} \underline{k}$$

$$\frac{\partial \phi}{\partial x} = 2x^2 \Rightarrow \phi = \frac{2}{3}x^3 + C_1(y,z)$$
$$\frac{\partial \phi}{\partial y} = \frac{\partial C_1}{\partial y} = 2yz$$
$$\Rightarrow C_1 = y^2 z + C_2(z)$$
$$\frac{\partial \phi}{\partial z} = y^2 + \frac{\partial C_2}{\partial z} \Rightarrow C_2 = C$$

a const.

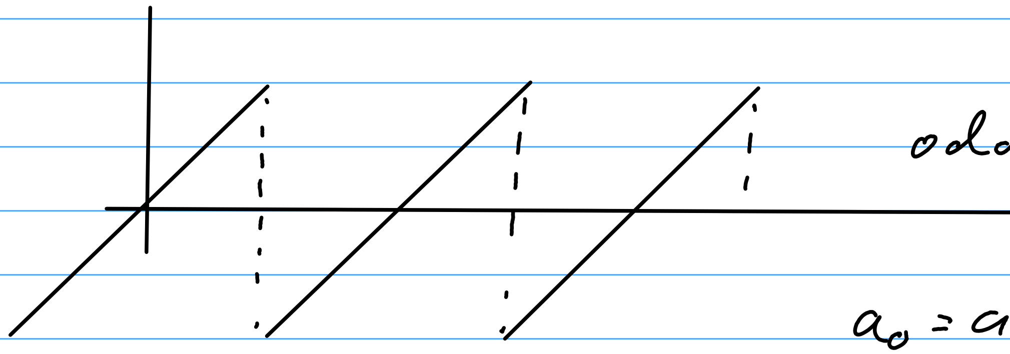
hence $\phi = \frac{2}{3}x^3 + y^2 z$ is a potential

$\phi(0,1,2) = 1 \qquad \phi(1,-1,7) = \frac{2}{3} + 7 = \frac{23}{3}$

$\Rightarrow \qquad I = 20/3$

q3 first part is from notes

$f(x) = x$ for $|x| < \pi$ & $f(x+2\pi) = f(x)$



odd

$a_0 = a_n = 0$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

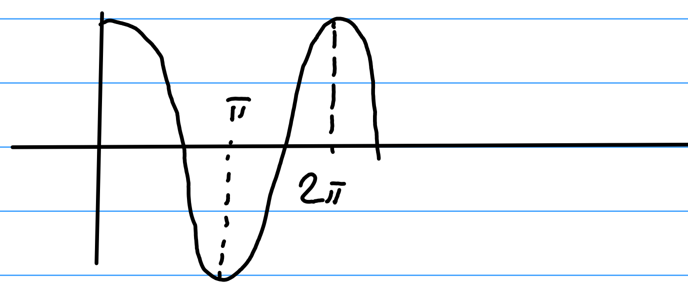
$$= \frac{1}{\pi}\int_{-\pi}^{\pi} x \sin nx \, dx$$

$\int u\,dv = uv\Big] + \int v\,du$

$$\pi b_n = -\frac{1}{n} x \cos nx \Big]_{-\pi}^{\pi} + \frac{1}{n}\int_{-\pi}^{\pi} \cos nx \, dx$$

$$= -\frac{2\pi}{n} \cos n\pi$$

$$= \frac{2\pi}{n} (-1)^{n+1}$$

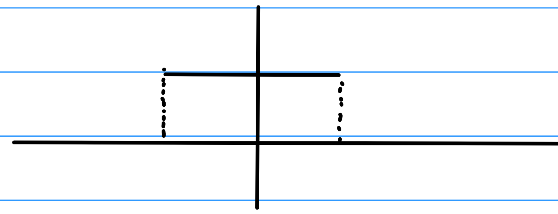


$$f(x) = \sum \frac{2\pi}{n} (-1)^{n+1} \sin nx$$

Q4

(a)

$$f(x) = \begin{cases} b & |x| < a \\ 0 & |x| \geq a \end{cases}$$



$$\tilde{f}(k) = \int_{-\infty}^{\infty} dx\, f(x)\, e^{-ikx}$$

$$= b \int_{-a}^{a} dx\, e^{-ikx}$$

$$= \frac{ib}{k} e^{-ikx} \Big]_{-a}^{a}$$

$$= \frac{ib}{k} \left(e^{-ika} - e^{ika} \right)$$

$$= \frac{2b}{k} \sin ka$$

(b) in the notes.