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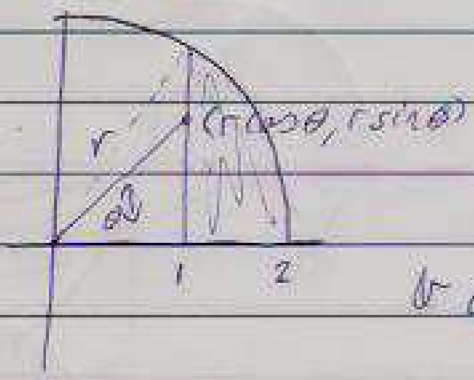
2006 q^1 cont.

$$R = \{(x,y) \mid x^2+y^2 \le 4,\ x \ge 1,\ y \ge 0\}$$



$r\cos\theta = 1$

$r = \frac{1}{\cos\theta}$

For θ_0 the max theta value is

$2 = \frac{1}{\cos\theta_0}$

$\cos\theta_0 = \frac{1}{2}$

$$\iint_R \frac{1}{\sqrt{x^2+y^2-1}}\,dx\,dy = \int_0^{\theta_0} d\theta \int_{1/\cos\theta}^{2} dr \frac{r}{\sqrt{r^2-1}}$$

the extra r is the jacobian

$$\int_{1/\cos\theta}^{2} \frac{r}{\sqrt{r^2-1}}\,dr \qquad \text{let } r = \sec\alpha$$

$\frac{d}{d\alpha}\sec\alpha = \frac{+\sin\alpha}{\cos^2\alpha}$

$= -\tan\alpha\, s$

$$= \int_{\theta}^{\cos^{-1}\frac{1}{2}} \frac{-\tan\alpha\sec\alpha}{\tan\alpha}\,d\alpha$$

$\cos^2\alpha + \sin^2\alpha = 1$

$-\tan^2\alpha = 1 - \sec^2\alpha$

$$= +\int_{\theta}^{\cos^{-1}\frac{1}{2}} \sec\alpha\,d\alpha$$

$$= \tan\theta_0 - \tan\theta$$

$\sec\alpha = 2$

$\alpha = \cos^{-1}\frac{1}{2} = \theta$

$\therefore$

$$\iint_R \frac{1}{\sqrt{x^2+y^2-1}}\,dx\,dy = \int_0^{\theta_0} d\theta\,(\tan\theta_0 - \tan\theta)$$

$$= \theta_0\tan\theta_0 + \log\cos\theta_0 = \frac{\sqrt{3}}{2}\cos^{-1}\frac{1}{2} - \log 2$$