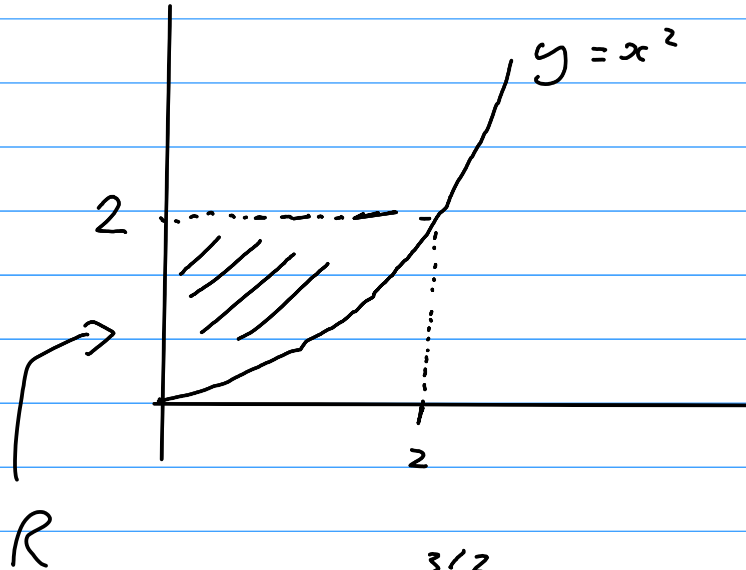
231 Annual Exam 2006 OUTLINE SOLNS — 28 May 2007

q1

$$R = \{ (x,y) \mid x^2 \le y \le 4 \}$$



$$\iint e^{y^{3/2}}\, dx\, dy$$

$$= \int_0^2 dx \int_{x^2}^4 dy\, e^{y^{3/2}}$$

This isn't going anywhere, try integrals in the other order

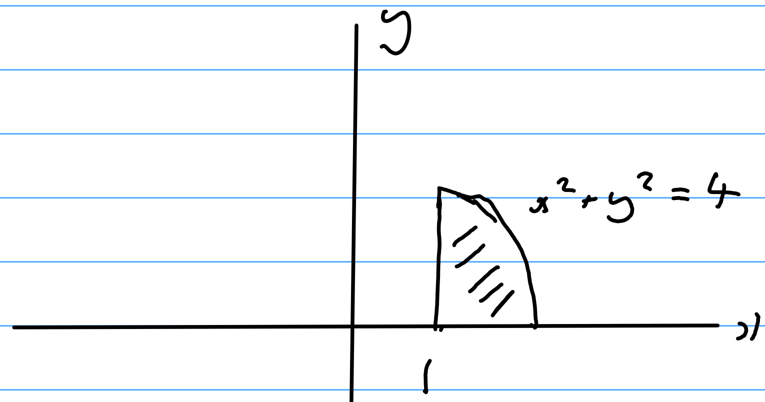
$$= \int_0^2 dy \int_0^{\sqrt{y}} dx\; e^{y^{3/2}}$$

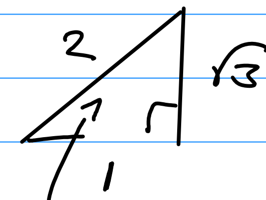
$$= \int_0^2 \sqrt{y}\, dy\, e^{y^{3/2}} \qquad u = y^{3/2}, \quad du = \frac{3}{2}\sqrt{y}\, dy$$

$$= \frac{2}{3} \int_0^{2^{3/2}} du\, e^u$$

$$= -\frac{2}{3} + \frac{2}{3}\, 2^{3/2} = \frac{2}{3}\left(2\sqrt{2} - 1\right)$$

(b) $R = \{(x,y) \mid x^2+y^2 \le 4,\ x \ge 1,\ y > 0\}$





$x = 1$

$r\cos\theta = 1$

$r = \sec\theta$

$\tan^{-1}\sqrt{3} = \theta_0$

$$I = \iint_R \frac{1}{\sqrt{x^2+y^2-1}}\,dx\,dy = \int_0^{\theta_0} d\theta \int_{\sec\theta}^{2} dr\, \frac{1}{\sqrt{r^2-1}}$$

$r = \sec u$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$r = \cos u^{-1}$

$$\frac{dr}{du} = \frac{-1}{\cos^2 u}(-\sin u)$$

$$= \tan u \sec u$$

$\sec u = 2$

$$\frac{d}{du}\tan u = \sec u$$

$$I = \int_0^{\theta_0} d\theta \int_\theta^{\theta_0} \sec u \, du$$

$$= \int_0^{\theta_0} (\tan\theta_0 - \tan\theta) \, d\theta$$

$$= \theta_0 \tan\theta_0 + \log\cos\theta_0$$

$$= \frac{\sqrt{3}}{2}\theta_0 + \log\frac{1}{2} = \frac{\sqrt{3}}{2}\cos^{-1}\frac{1}{2} - \log 2$$