231 Christmas quiz 2007[1]

6 December 2007

1. What is
$$\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \int_0^{\infty} dr r^2 \sin\theta e^{-r^2}? \tag{1}$$
Well
$$d\phi d\theta dr r^2 \sin\theta = dV = dxdydz \tag{2}$$
and the limits give all of $\mathbf{R}^3$, so
$$\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \int_0^{\infty} dr r^2 \sin\theta e^{-r^2} = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz e^{-x^2-y^2-z^2} \tag{3}$$
$$= \left(\int_{-\infty}^{\infty} e^{-x^2}\right)^3 = \pi^{3/2} \tag{4}$$

2. When was the college founded? 1592.

3. What is
$$\int_0^{2\pi} \cos^6\theta? \tag{5}$$
Well use
$$\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} \tag{6}$$
so by the binomial theorem
$$\cos\theta = \frac{1}{2^6}(\ldots + 20 + \ldots) \tag{7}$$
where we have left out the terms of the form $\exp in\theta$ for integer n, these will all integrate to zero, hence, we integrating, we have
$$\int_0^{2\pi} \cos^6\theta = \frac{5\pi}{8} \tag{8}$$

4. What first is associated with Kenneth Tynan? It is probably best you look this up on Wikipedia.

5. So say $\mathbf{F} = (x, y, z)$ and $\phi = xyz$, let $\hat{\mathbf{n}}$ be the unit normal to the surface $\phi = 1$, what is $\mathbf{F}\cdot\hat{\mathbf{n}}$ at $x = y = z = 1$? Well the key thing is that the normal to a surface ϕ =const is given by the gradient of ϕ, so here the normal is (yz, xz, xy), at $x = y = z = 1$ this is $(1, 1, 1)$ and normalizing it gives $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$. Dotting this with $\mathbf{F}$ at $x = y = z = 1$ gives $\sqrt{3}$.

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6. Who won this years nobel prize in literature? Doris Lessing.

7. What is
$$\int_C \mathbf{F}\cdot\mathbf{dl} \tag{9}$$
for C the semi-circle of unit radius in the $y > 0$ half plane about the origin from $x = 1$ to $x = -1$ and
$$\mathbf{F} = \frac{y}{x^2+y^2}\mathbf{i} - \frac{x}{x^2+y^2}\mathbf{j} \tag{10}$$
Well, as we saw in PS6 $\mathbf{F} = -\nabla\theta$, the polar angle and, we have seen before that for a conservative field
$$\int_C \mathbf{F}\cdot\mathbf{dl} = \phi(b) - \phi(a) \tag{11}$$
where the curve goes from a to b, here, this gives the answer $-\pi$.

8. In *Little Miss Sunshine* Steve Carell plays the second most distinguished expert on whom? Proust.