231 Christmas quiz 2007^1

6 December 2007

1. What is

$$\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \int_0^{\infty} dr r^2 \sin \theta e^{-r^2}?$$
(1)

Well

$$d\phi d\theta dr r^2 \sin \theta = dV = dx dy dz \tag{2}$$

and the limits give all of \mathbf{R}^3 , so

$$\int_{0}^{2\pi} d\phi \int_{0}^{\pi} d\theta \int_{0}^{\infty} dr r^{2} \sin \theta e^{-r^{2}} = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz e^{-x^{2}-y^{2}-z^{2}}$$
(3)

$$= \left(\int_{-\infty}^{\infty} e^{-x^2}\right)^3 = \pi^{3/2} \tag{4}$$

- 2. When was the college founded? 1592.
- 3. What is

$$\int_0^{2\pi} \cos^6 \theta? \tag{5}$$

Well use

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \tag{6}$$

so by the binomial theorem

$$\cos \theta = \frac{1}{2^6}(\dots + 20 + \dots)$$
 (7)

where we have left out the terms of the form $\exp in\theta$ for integer n, these will all integrate to zero, hence, we integrating, we have

$$\int_{0}^{2\pi} \cos^{6} \theta = \frac{5\pi}{8}$$
 (8)

- 4. What first is associated with Kenneth Tynan? It is probably best you look this up on Wikipedia.
- 5. So say $\mathbf{F} = (x, y, z)$ and $\phi = xyz$, let $\hat{\mathbf{n}}$ be the unit normal to the surface $\phi = 1$, what is $\mathbf{F} \cdot \hat{\mathbf{n}}$ at x = y = z = 1? Well the key thing is that the normal to a surface $\phi = \text{const}$ is given by the gradient of ϕ , so here the normal is (yz, xz, xy), at x = y = z = 1 this is (1, 1, 1) and normalizing it gives $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$. Dotting this with \mathbf{F} at x = y = z = 1 gives $\sqrt{3}$.

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- 6. Who won this years nobel prize in literature? Doris Lessing.
- 7. What is

$$\int_{C} \mathbf{F} \cdot \mathbf{dl} \tag{9}$$

for C the semi-circle of unit radius in the y > 0 half plane about the origin from x = 1 to x = -1 and

$$\mathbf{F} = \frac{y}{x^2 + y^2} \mathbf{i} - \frac{x}{x^2 + y^2} \mathbf{j}$$
(10)

Well, as we saw in PS6 $\mathbf{F} = -\nabla \theta$, the polar angle and, we have seen before that for a conservative field

$$\int_{C} \mathbf{F} \cdot \mathbf{dl} = \phi(b) - \phi(a) \tag{11}$$

where the curve goes from a to b, here, this gives the answer $-\pi$.

8. In *Little Miss Sunshine* Steve Carell plays the second most distinguished expert on whom? Proust.