

1S1 Tutorial Sheet 1: Solutions¹

22 October 2006

Questions

1. (2) What is the natural domain of

$$f(x) = \sqrt{-x^2 + 4x - 3} \quad (1)$$

where you can note the factorization: $-x^2 + 4x - 3 = -(x-1)(x-3) = (x-1)(3-x)$.

Solution: The natural domain covers all the places where the function is real. The function here is real provided the thing in the square root is positive. Now, we can factorize

$$-x^2 + 4x - 3 = -(x-1)(x-3) = (x-1)(3-x) \quad (2)$$

and this is positive if $x \geq 1$ and $x \leq 3$. Put another way, the quadratic is zero when $x = 1$ or $x = 3$, you can get this using the usual minus b plus or minus the square root of b squared minus four ac all over two a formula. Now it can only change sign at these two places and you can see for super large positive or negative x it is negative, hence, it must be positive between $x = 1$ and $x = 3$. Either way, the domain is $1 \leq x \leq 3$.

2. (2) We define the function

$$f(x) = x^2 \quad (3)$$

on the domain $x \geq 0$. Is this function invertible? If so, what is its inverse?

Solution: So, the function is invertible if it is strictly increasing or decreasing and x^2 is strictly increasing for $x > 0$, hence, this is an invertible function. To invert it

$$y = x^2 \quad (4)$$

hence

$$x = \sqrt{y} = f^{-1}(y) \quad (5)$$

or $f^{-1}(x) = \sqrt{x}$. This is real for $x \geq 0$.

3. Consider the piecewise function

$$f(x) = \begin{cases} x & x < 2 \\ 3 & x = 2 \\ 1 & 2 < x \leq 3 \\ 4 - x & x > 3 \end{cases} \quad (6)$$

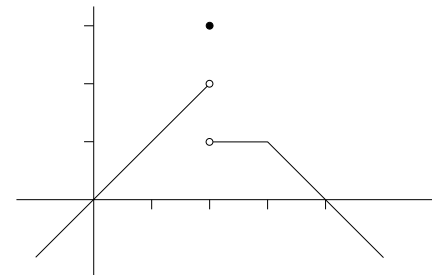
¹Conor Houghton, houghton@maths.tcd.ie, see also <http://www.maths.tcd.ie/~houghton/1S1>

- (a) (2) Graph the function.

- (b) (2) Calculate

$$\lim_{x \rightarrow 2^-} f(x), \quad \lim_{x \rightarrow 2^+} f(x), \quad f(2), \quad \lim_{x \rightarrow 3} f(x)$$

Solution: So, again, the stuff on the right hand of the curly bracket gives a list of instructions as to what $f(x)$ is for different intervals. It looks like this



and looking that this graph is is clear that

$$\lim_{x \rightarrow 2^-} f(x) = 2, \quad \lim_{x \rightarrow 2^+} f(x) = 1, \quad f(2) = 3 \quad \lim_{x \rightarrow 3} f(x) = 1 \quad (8)$$

Extra Questions

1. What is the natural domain of

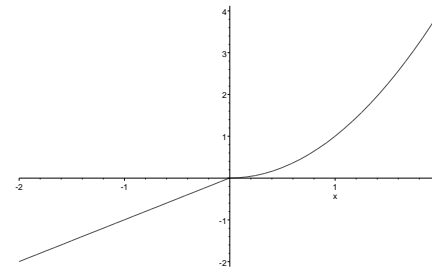
$$f(x) = \sqrt{-x} \quad (9)$$

Solution: Again, $f(x)$ if what is inside the square root sign is real, which means $x \leq 0$.

2. Graph and invert

$$f(x) = \begin{cases} x^2 & x > 0 \\ x & x < 0 \end{cases} \quad (10)$$

Solution: Well the graph looks like



and because the function is defined peicewise we invert is piecewise using the same method for each bit as before to get

$$f^{-1}(x) = \begin{cases} \sqrt{x} & x > 0 \\ x & x < 0 \end{cases} \quad (11)$$

3. Find

$$\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1} \quad (12)$$

Solution: So, it seems that the function is not defined at $x = 1$, at $x = 1$ but numerator and denominator are zero, however, if we factorize that top we find

$$f(x) = \frac{x^2 + x - 2}{x - 1} = \frac{(x - 1)(x + 2)}{x - 1} = x + 2 \quad (13)$$

provided $x \neq 1$, hence

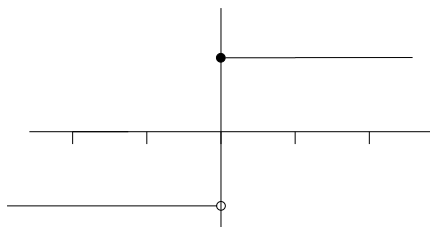
$$\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1} = 3 \quad (14)$$

4. Graph the function

$$f(x) = \frac{|x|}{x} \quad (15)$$

What is its domain? What are the one-sided limits at $x = 0$?

Solution: So, $|x| = x$ for $x \geq 0$ and $-x$ for $x < 0$, hence the function is zero over zero at $x = 1$ and is not define. Otherwise, it is plus one for $x > 0$ and minus one for $x < 0$ and the graph looks like



It is easy to see

$$\lim_{x \rightarrow 0-} f(x) = -1, \quad \lim_{x \rightarrow 0+} f(x) = 1 \quad (16)$$