1S1 Tutorial Sheet 5[1]

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Useful facts:

- The main property of the **exponential** is

$$\frac{d}{dt}e^t = e^t \tag{1}$$

and $e = 2.71828183$.

- The Taylor expansion of the exponential can easily be calculated because the exponential can be differentiated easily and

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \cdots + \frac{1}{n!}x^n + \cdots \tag{2}$$

- The inverse of the exponential is the **natural log**, $\ln x = \log_e x$

$$\ln e^x = x \tag{3}$$

- **Differenciating the natural log**:

$$\frac{d}{dx}\ln x = \frac{1}{x} \tag{4}$$

- **Implicit differentiation**: If y is related to x by an equation, often the easiest way to find dy/dx is to differentiate the equation and then solve for dy/dx.

- **l'Hôpital**: If $f(a) = g(a) = 0$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)} \tag{5}$$

hence, unless $f'(a)$ and $g'(a)$ are both zero

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)} \tag{6}$$

[1] Conor Houghton, `houghton@maths.tcd.ie`, see also `http://www.maths.tcd.ie/~houghton/1S1`

Questions

The numbers in brackets give the numbers of marks available for the question.

1. (4) Find the first four terms in the Taylor expansion of $\ln(x)$ about $x = 1$.
2. (2) Use implicite differentiation to find dy/dx where
$$\ln x + 2xy + y^2 = 0 \tag{7}$$
3. (2) Find
$$\lim_{x\to 1} \frac{x^2-1}{x^2-x} \tag{8}$$
both by factorizing and by l'Hôpital's rule.

Extra Questions

The questions are extra; you don't need to do them in the tutorial class.

1. Find df/dx where $f(x) = \cos^{-1} x$.
2. Find dy/dx where $\ln y + \ln x = \exp xy$.
3. Differentiate $\ln \cos x$.
4. Differentiate $\ln x^3$ both by the chain rule and by $\ln x^3 = 3 \ln x$.
5. Find
$$\lim_{x\to 0} \frac{e^x - 1}{\sin x} \tag{9}$$
6. Find
$$\lim_{x\to 1} \frac{\ln x}{1-x} \tag{10}$$
7. Find
$$\lim_{x\to 1} \frac{\ln x}{\exp(x-1)} \tag{11}$$
8. Find
$$\lim_{x\to 1} \frac{x^2-1}{x^2-2x+1} \tag{12}$$